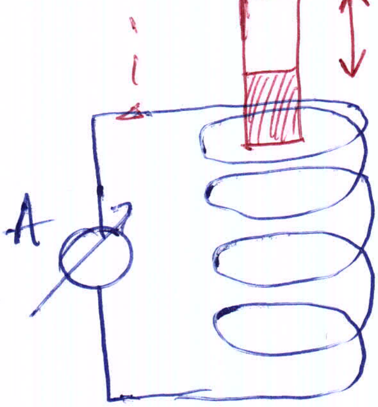
ELECTROMAGNETIC INDUCTION

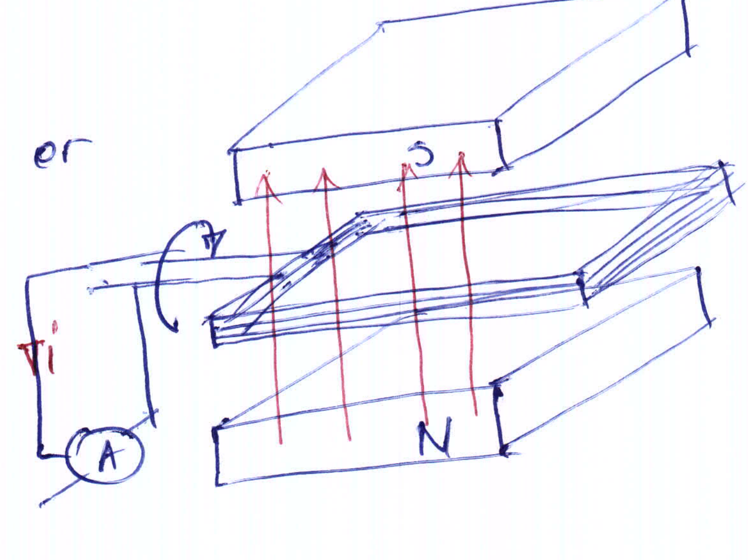
If the magnetic field changes through a circuit, and consequently the magnetic flux, an electromotive force (emf) and a current is induced in the circuit. This phenomenon is called electromagnetic induction whose central principle is described by the Faraday's law. The direction of the induced current will be described by the Lentz law. These principles will help us to understand electrical energy conversion devices such as motors, generators, transformers.

Electromagnetic induction tells us that a time varying magnetic field can act as a source of electric field. We will also see that a time varying electric field can act as a source of magnetic field. These would lead to a package of formulas: the Maxwell's equations that describe the behavior of electric and magnetic fields in any situations. They pave the way for understanding electromagnetic waves, the topic of a next chapter.

1 Induction experiments



or



rotating coil in a magn. field $\vec{B}$

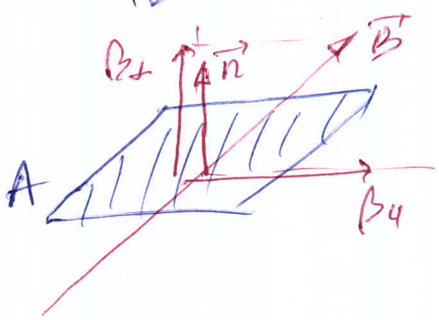
$\Downarrow$

I

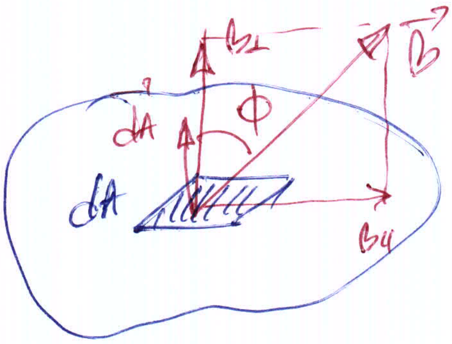
In both experiments a current appear when varying the magnetic flux Φ_B through the coil connected to the galvanometer.

② Faraday's law

Magnetic flux: $\Phi_B = \vec{B}\cdot\vec{A} = B_\perp A$



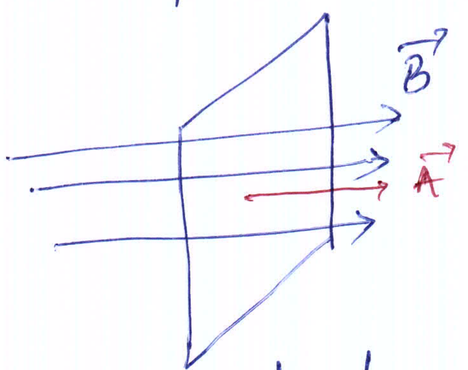
Through a complex surface, we discretize in area elements

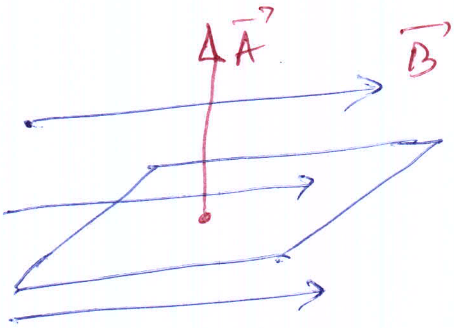


$$\boxed{\Phi = \oiint \vec{B}\cdot\vec{dA} = \oiint B\,dA\cos\phi}$$

$$d\Phi_B = \vec{B}\cdot\vec{dA} = B_\perp dA$$
$$= B\,dA\cos\phi$$

Examples





$\vec{B} \parallel \vec{A} \Rightarrow \Phi = \Phi_{max} = BA$

$\vec{B} \perp \vec{A} \Rightarrow \Phi = 0 \quad (\cos\phi = 0)$

for arbitrary ϕ angle position Φ varies as $\cos\phi$ between Φ_{max} and 0

Faraday's law of induction

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

The induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop

Direction of induced emf

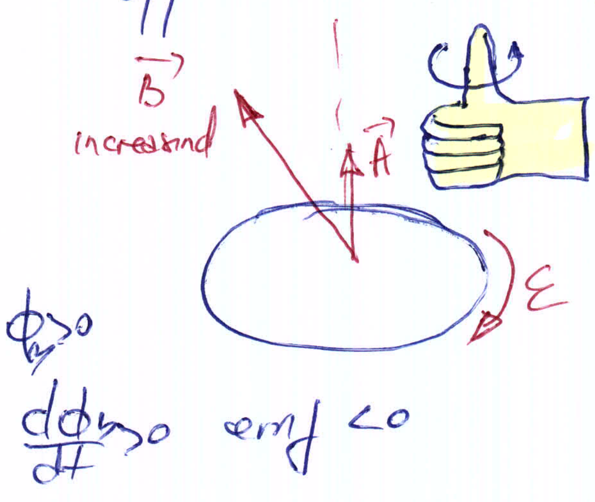
Rules:

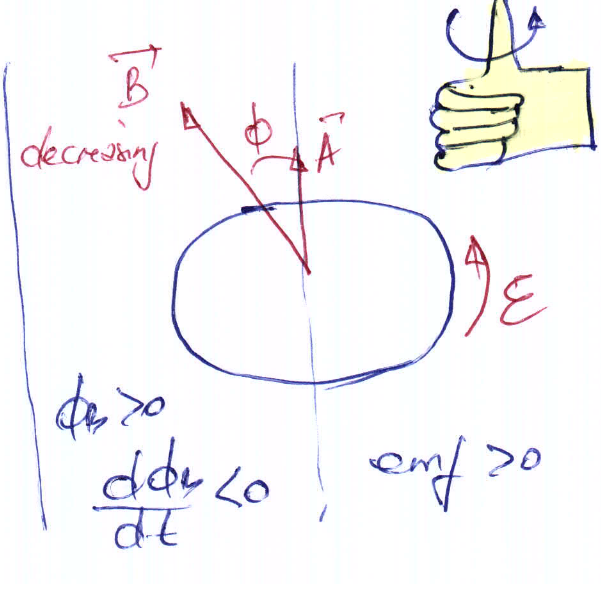
① Define a positive direction for the vector area $\vec{A}$

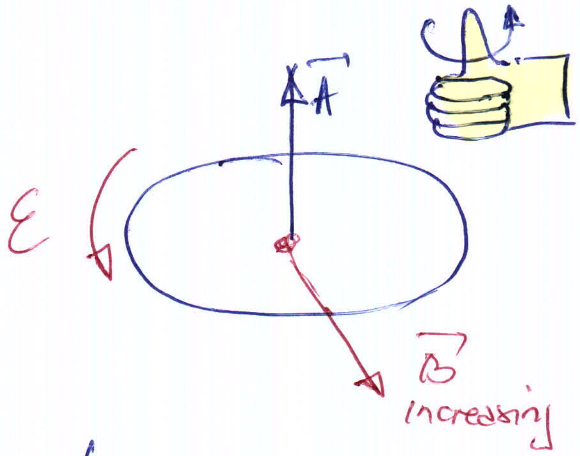
② From the direction of $\vec{A}$ and $\vec{B}$ determine the sign of Φ_B and $\frac{d\Phi_B}{dt}$

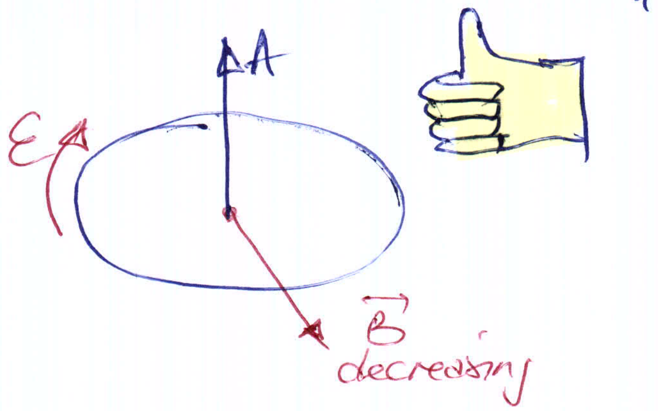
③ Determine the sign of the induced emf and current.
If the flux is increasing $\frac{d\Phi_B}{dt} > 0$, then emf is negative
decreasing $\frac{d\Phi_B}{dt} < 0$, then emf > 0

④ Finally, determine the direction of induced emf or current using the right hand. Curl the fingers of right hand around $\vec{A}$ with thumb indicating $\vec{A}$. If the induced emf is positive it is in same direction as your fingers; if induced emf is negative; it is in opposite direction of fingers.









$\Phi_B < 0$

$\frac{d\Phi_B}{dt} < 0$ (more negative)

$\Rightarrow$ Emf > 0

$\Phi_B < 0$

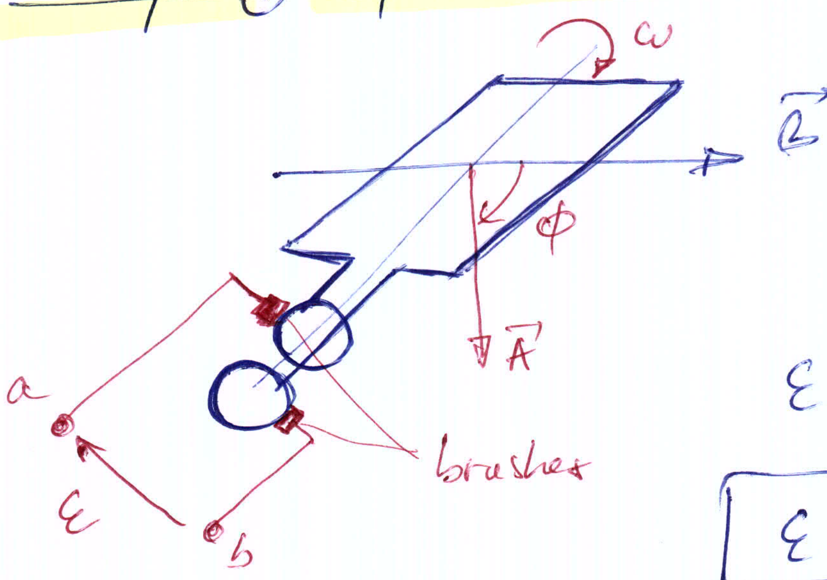
$\frac{d\Phi_B}{dt} > 0$ (less negative)

Emf < 0

Obs: For a coil with N identical turns

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

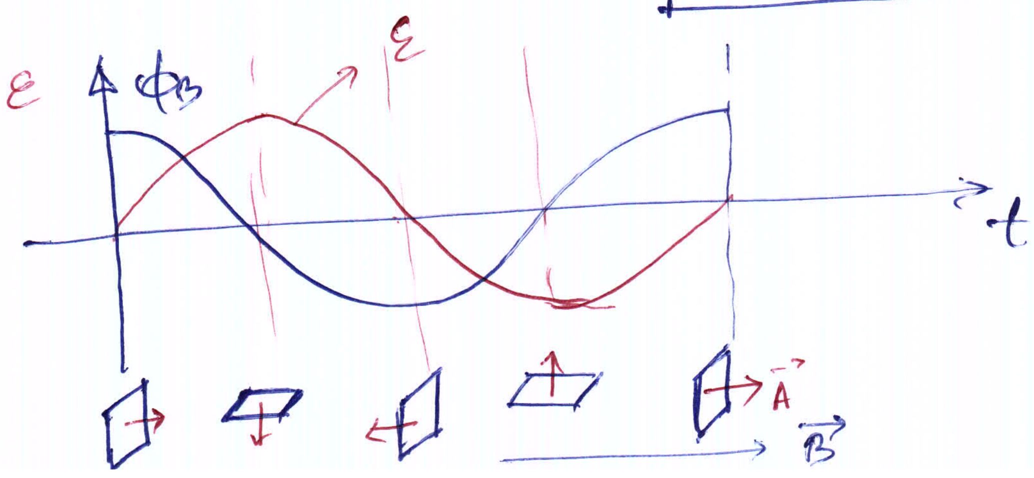
Example ① Simple alternator



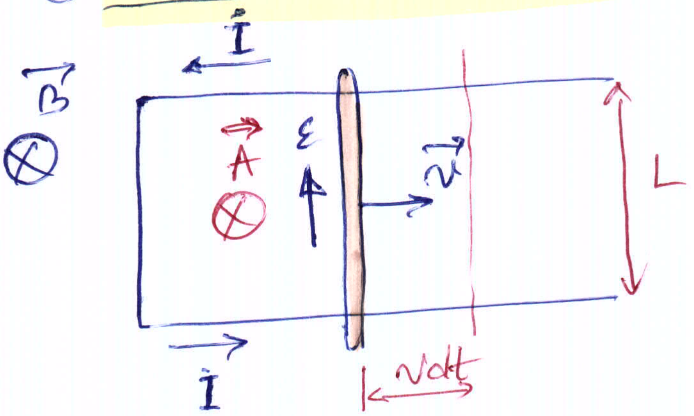
$\phi = \omega t$

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA\cos\omega t)$$

$$\boxed{\varepsilon = \omega BA\sin\omega t}$$



② Slidewire generator



$\vec{B} \parallel \vec{A}$

$\phi_B = BA$

$\mathcal{E} = -\frac{d\phi_B}{dt} = -B\frac{dA}{dt}$

$dA = v\,dt\,L$

$\Rightarrow \mathcal{E} = -BLv\frac{dt}{dt}$

$\boxed{\mathcal{E} = -BLv}$

means that $\mathcal{E}$ is counterclockwise in the loop

Obs $|\vec{v}| = ct \Rightarrow \mathcal{E} = ct$
$\Rightarrow$ DC generator.

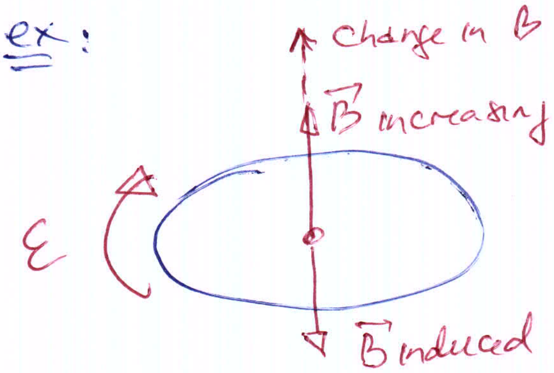
③ LENZ law

$\rightarrow$ convenient alternative to determine the direction of Emf/current.

The direction of any magnetic induction effect is such to oppose the cause of the effect.

$\rightarrow$ directly related to energy conservation

ex:



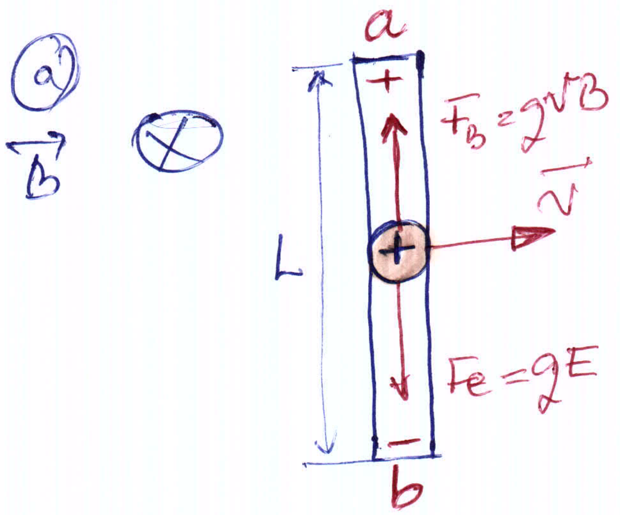
By Lentz law the induced current must produce a magnetic field downwards $\vec{B}_{induced}$ which opposes to the change in flux due to increasing $\vec{B}$

(4) Motional electromotive force

When a conductor moves in a magnetic field (as in generators) we can gain additional insight on the origin of induced emf by considering the magnetic forces on the mobile charges in conductor (Lorentz)

conducting rod moving in a uniform field $\vec{B}$

(a)



charges in the moving rod are acted by the magnetic force F_B upwards

and

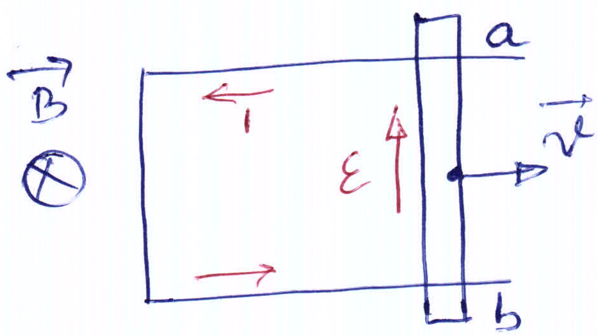
create a charge separation which induces an electric field and electric force F_e

at equilibrium: $F_B = F_E \Rightarrow$

$$q\,vB = qE \Rightarrow E = vB$$

$$V_{ab} = V_a - V_b = EL = vBL$$

(b) The moving rod is connected to a stationary U shaped conductor



No magnetic force acts on the charges in the static conductor. The moving conductor become a source of emf $\mathcal{E} = V_{ab}$

$$\Rightarrow \boxed{\mathcal{E} = vBl} = iR$$

resistance of static conductor.

Motional emf : General form

We can generalize the concept of motional emf for a conductor with arbitrary shape moving in a magnetic field. For an element $\vec{dl}$ the contribution $d\mathcal{E}$ to emf is:

$$d\mathcal{E} = (\vec{v} \times \vec{B}) \cdot \vec{dl}$$

$$\Rightarrow \boxed{\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot \vec{dl}}$$

5 Induced electric fields

When a conductor moves in a magnetic field, we can understand the induced emf on the basis of magnetic forces on charges on conductor. But, an induced emf also occur when there is a changing flux through a stationary conductor. What is that pushes the charges around the circuit in this situation?

There has to be an induced electric field in the conductor caused by the changing magnetic flux.

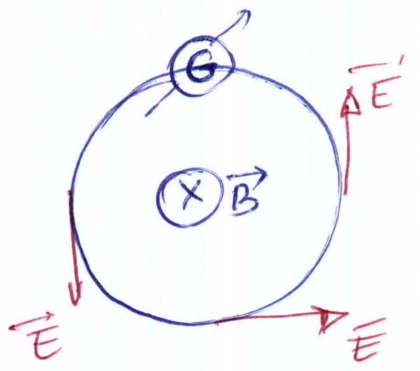
$\Rightarrow$ new source of electric field (beyond standard charges) which is changing magnetic field. This new field is **not conservative**, because the integral of a closed path is non-zero but equal to the induced emf:

$$\boxed{\oint \vec{E} \cdot \vec{dl} = \mathcal{E}}$$

From Faraday law: $\boxed{\oint \vec{E} \cdot \vec{dl} = -\frac{d\Phi_B}{dt}}$

$\vec{E}$ = non-conservative field called NON-ELECTROSTATIC field

Example Stationary circular loop:



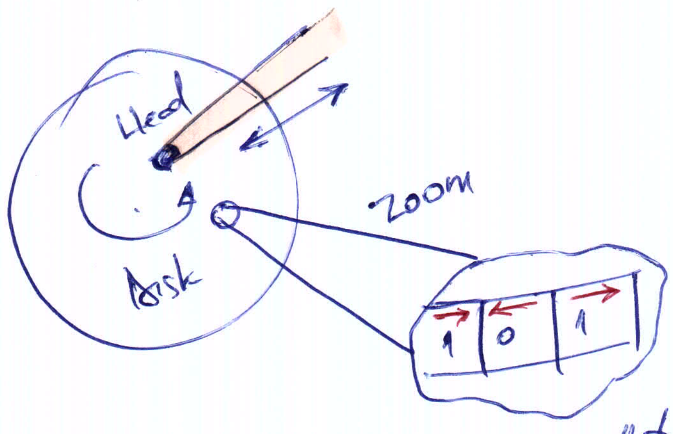
$$\oint \vec{E} \cdot d\vec{l} = 2\pi r E = -\frac{d\phi_B}{dt}$$

$$\Rightarrow E = \frac{1}{2\pi r}\left|\frac{d\phi_B}{dt}\right|$$

Obs Even if $\vec{E}$ generated by $\frac{d\phi_B}{dt}$ is non-conservative and non-electrostatic the effect on a charge q will be same as conservative $\vec{E}$, so that a force $\vec{F} = q\vec{E}$ will be exerted.

Applications of induced emf (induced electric fields)

a) read-heads of hard disks (inductive) : coils placed on movable arms placed in proximity of a spinning disk where datas (1) (0) are stored magnetically.



The head-coil feels changes of ϕ_B when passing domains (1) and (0) ($\vec{B}$ varies) which are "translated" in emf

b) generators

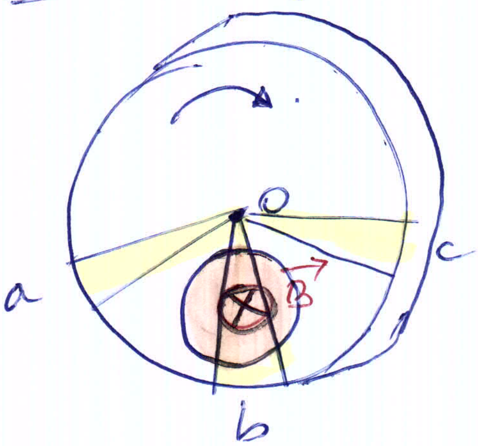
6 Eddy Currents

(Foucault) discovered by the Frech scientis Leon Foucault (1851)

RO: curenți turbionari

In the examples analised before the induced currents have been confined in well defined paths in conductors. However many pieces of electrical equipment contain masses of metal moving in magnetic fields or located in changing magnetic fields. In this case, we have induced currents that circulate in the volume of the material along swirling paths likewise swirling eddies in a river ⟹ EDDY CURRENTS

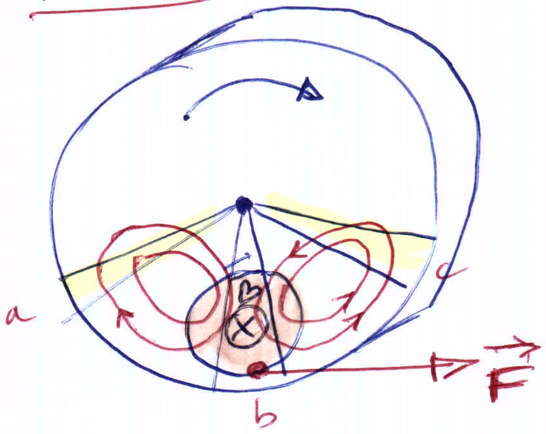
ex: rotating metal disk in a magnetic field confined in a limited disk area



Sector moving in the field (ob) will have an emf induced on it.

Adjacent sectors (oa) and (ob) are not in the field but they provide return conducting paths for charges ⟹ eddy currents.

The interaction between this eddy currents and the field causes a force opposing rotation ⟹ BRAKING FORCE (from Lentz rule)



Applications of Eddy currents

→ rapid brakes of rotating metallic disks (stopping a circular saw when power is turned off)

→ in sensitive ballances for damping vibrations

→ shiny metal disk in power consumption meters rotates due to eddy currents induced in the disk by magnetic fields caused by sinusoidally varying current in coil.

→ induction furnaces

→ metal detectors used in airports security checkpoints operate by detecting eddy currents induced in metallic objects.

Undesirable effects

→ waste of energy in alternating-current transformers by Joule effect of induced eddy-currents. To minimize this, the core is designed to minimize the width of the eddy current paths.

(7) Displacement current and Maxwell's equations

We have seen that a varying magnetic field gives rise to an induced electric field. In one of the most remarkable examples of symmetry of nature, it turns out that a varying electric field gives rise to a magnetic field. This effect is of huge importance because it explain the existance of radio waves, gamma rays, visible light and any other forms of electromagnetic waves.

Generalizing the Ampère's law

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$ is incomplete

A time varying electric field generate a displacement current i_D which acts as a source of magnetic field exactly as a conducting current.

$$\boxed{i_D = \varepsilon \frac{d\Phi_E}{dt}}$$

ε = material permittivity.

Maxwell's equations of electromagnetism

We can wrap-up in a single package all of the relationship between electric and magnetic fields and their sources

$\Rightarrow$ four equations called Maxwell's equations

(1) $$\boxed{\oiint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}}$$ Gauss's law for $\vec{E}$ fields

(2) $$\boxed{\oiint \vec{B}\cdot d\vec{A} = 0}$$! no mgn. monopoles. Gauss's law for $\vec{B}$ fields

(3) $$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0\left(i_c + \varepsilon_0 \frac{d\Phi_E}{dt}\right)}$$ Ampères law including displacement currents

(4) $$\boxed{\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}}$$ Faraday's law

integrals on stationary closed path.

Signification

(1) $$\boxed{\oint\!\!\oint \vec{E}\, d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}}$$

$\Rightarrow$ enclosed charges in a surface are source of conservative electric fields

(2) $$\boxed{\oint\!\!\oint \vec{B}\, d\vec{A} = 0}$$

$\Rightarrow$ no magnetic monopoles (single magn charges) to act as sources of magnetic field.

(3) $$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 \left(i_c + \varepsilon_0 \frac{d\Phi_E}{dt} \right)_{encl}}$$

$\Rightarrow$ both conduction currents and displacement currents $\varepsilon_0 \frac{d\Phi_E}{dt}$ act as sources of magnetic field.

(4) $$\boxed{\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}}$$

$\Rightarrow$ a changing magnetic field induces an electric field, the line integral on closed stationary path is not-zero $\Rightarrow$ $\vec{E}$ produced by changing magnetic flux is non-conservative

<u>Obs</u>: The total $\vec{E}$ in a point of space can be a superposition between $\vec{E_c}$ caused by a distribution of charges in rest and a magnetically induced non-electrostatic field $\vec{E_n}$

$$\boxed{\vec{E} = \vec{E_c} + \vec{E_n}}$$

conservative $\oint \vec{E}\, d\vec{l} = 0$

$\vec{E_c}$ does not contribute on integral path in Faraday's law

$\vec{E_n}$ - does not contribute to Gauss's law $\oint\!\!\oint \vec{E}\, d\vec{A} = 0$

Symmetry in Maxwell's equations

In empty space: (no charges $\Longrightarrow Q_{encl} = 0$)

$$\oint\!\!\oint \vec{E} \cdot d\vec{A} = 0 \quad \text{and} \quad \oint\!\!\oint \vec{B} d\vec{A} = 0$$ identical

no conduction current $i_C = 0$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \longleftrightarrow \oint \vec{E} \cdot dl = -\frac{d\Phi_B}{dt}$$

identical form apart a constant and sign.

if: $\Phi_E = \oint\!\!\oint \vec{E} \cdot d\vec{A}$

$\Phi_B = \oint\!\!\oint \vec{B} \cdot d\vec{A}$

$$\Rightarrow \oint \vec{B} d\vec{l} = \mu_0 \varepsilon_0 \frac{d}{dt} \oint\!\!\oint \vec{E} \cdot d\vec{A}$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

Conclusion from Maxwell's eq we see that:

A time varying field of either kind induces a field of the other kind in neighboring regions of space.

Maxwell recognized that these relationships predict the existance of electromagnetic disturbances consisting of time varying electric and magnetic fields that can propagate from one region of space to another, even if no matter is present. Such disturbances are called ELECTROMAGNETIC WAVES provide the physical basis for light, radio-TV waves, IR, UV, X and all the rest of electromagnetic spectrum.

All the basic relationship between fields and their sources are contained in Maxwell's equations. We can derive Coulomb's law from Gauss law, Biot and Savart from Ampère's law, etc.

When we add the equation that defines $\vec{E}$ and $\vec{B}$ in terms of the forces they exert on a charge q, namely:

$$\boxed{\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)}$$

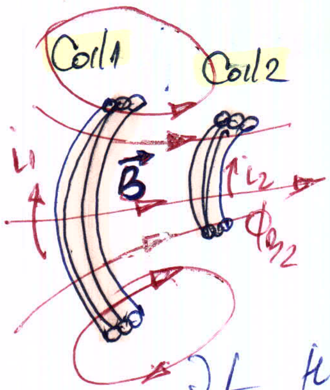
we have all the fundamental relationships of electromagnetism!

Maxwell's eq. for electromagnetisms are likewise Newton's laws for mechanics, thermodinamics law for thermodynamics

INDUCTANCE

(1) Mutual inductance

When a changing current i_1 in one circuit causes a changing magnetic flux in a second circuit an emf $\mathcal{E}_2$ is induced in the second circuit. Likewise, a second changing i_2 in second circuit induces an emf in first circuit.



$$\boxed{\mathcal{E}_2 = -M\frac{di_1}{dt}} \qquad \boxed{\mathcal{E}_1 = -M\frac{di_2}{dt}}$$

M = mutual inductance

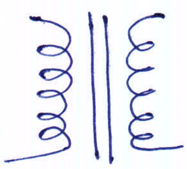
If the circuits are coils of wire with N_1 and N_2 turns, the mutual inductance is:

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2}$$

$$[M]_{SI} = H \text{ (Henry)} = 1\frac{Wb}{A} = 1\frac{V s}{A} = 1\,\Omega\cdot s$$
$$= 1\,J/A^2$$

Henry is a rather large unit of mutual inductance; typical values are mH, μH, ...

Applications: AC transformers, the value of M depends on the value of the geometry



primary secondary

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1} \longrightarrow \frac{V_2}{V_1} = \frac{N_2}{N_1}$$

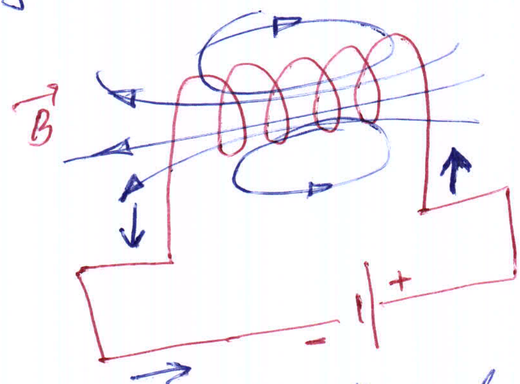
② Self-inductance

A changing current i in any circuit causes a self induced emf $\mathcal{E}$. The inductance (or self inductance) L depends on the geometry of the circuit and the material surrounding it

$$\boxed{\mathcal{E} = -L\frac{di}{dt}}$$

The inductance of a coil of N turns is related to the average flux Φ_B through each turn caused by the current i in the coil

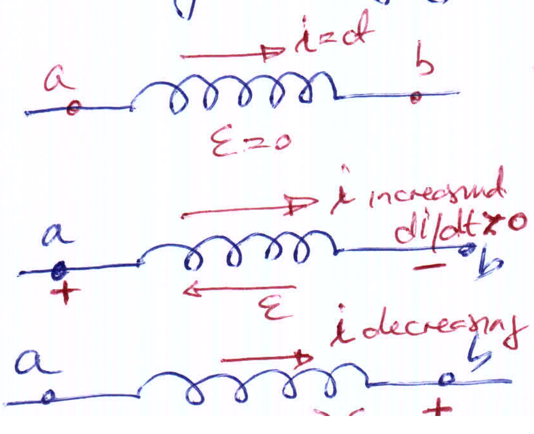
$$\boxed{L = N\frac{\Phi_B}{i}}$$



An inductor is a circuit device, usually including a coil of wire, intended to have a substantial inductance

L

Obs : The potential difference across a resistor R depends on the current I ($V = Ri$). The potential difference across an inductor depends on the rate of changing current



$V_{ab} = L\frac{di}{dt} = 0$

$V_{ab} = L\frac{di}{dt} > 0$

$V_{ab} = L\frac{di}{dt} < 0$

Self induced emf opposes changes in current.

(3) Magnetic field energy

An inductor with inductance L carrying current i has energy U associated to the inductor's magnetic field

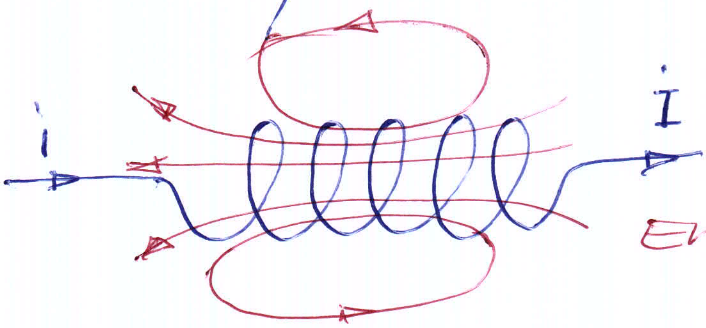
$$\boxed{U = \frac{1}{2} L i^2}$$

Obs: A resistor is a device in which energy is irrecoverably dissipated. By contrast, the energy stored in a current carrying inductor can be recovered when the current decreases to zero.

Magnetic energy density u (energy / unit volume)

$$u = \frac{B^2}{2\mu_0} \quad \text{(in vacuum)}$$

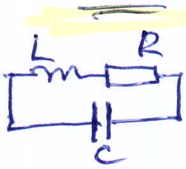
$$U = \frac{B^2}{2\mu} \quad \text{(in a material with permitivity } \mu)$$



Energy density $u = \frac{B^2}{2\mu_0}$ ✓

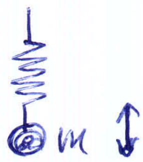
Stored energy $U = \frac{1}{2} L i^2$

Obs: Reanalize analogy with damped oscillator (see seminary 1st term).



$L \longleftrightarrow m$

inertia



$\longleftrightarrow$ opposes changement of

a: $\longleftrightarrow$ x

In mechanics (harmonic oscillator) the mass m stores energy as kinetic energy:

$$K = \frac{1}{2} m v^2$$

Here, the inductance stores <u>magnetic field energy</u>:

$$U = \frac{1}{2} L i^2$$

(see analogy

$$q \longleftrightarrow x$$

$$i = \frac{dq}{dt} \longrightarrow v = \frac{dx}{dt}.$$